Absolute Humidity from Relative Humidity

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Abstract

A set of python functions are described that convert relative humidity ($0 - 100\%$) to absolute humidity (kg of water per kg of dry air), volumetric humidity (g of water per m^3 of air), and dew point (°C) for water-air systems in meteorlogical temperature and pressure ranges. An implementation using a `BME280` pressure and temperatrure sensor is described.

1 Background

Humidity of ambient air is an important factor for human comfort. As of 2021, low-cost humidity measurement electronic modules are often integrated into smartphones, personal wristwatches (see “ABC” or “altitude, barometric, compass” watches), and electric thermometers. The default units of humidity displayed by such devices is often “RH” or “relative humidity”. Relative humidity of air could be described as how much air’s ability to absorb water vapor is occupied. Relative humidity of air, usually expressed as a percentage, is a dimensionless ratio of the current value of the partial pressure of water vapor to its theoretical maximum value. However, this theoretical maximum capacity varies depending on air temperature; for example, if a room in a typical human dwelling is sealed (by closing doors and air vents) and then heated (with a space heater), the relative humidity of the room’s air will fall despite no water vapor leaving the system.

1.1 Definitions and Notation

Most equations used in this work use notation used in [2, Sonntag (1994)]. Some equations use notation used in [1, Green (2008)]. All equations are accompanied by variable definitions for clarity. Note that equations from [1, Green (2008)] usually have consistent units while equations from [2, Sonntag (1994)] require careful inspection of assumed units of variables and constants.

1.2 Conversion Equations

1.2.1 Sonntag-1990 equation for saturation vapor pressure of water e_w

$$\begin{aligned} \ln\left(e_{\mathrm{w}}(T)\right) &= -6096.93\,T^{-1} + 21.2409642 - 2.711193\times10^{-2}\,T \\ &\quad + 1.673952\times10^{-5}\,T^{2} + 2.433502\ln(T) \qquad (1) \end{aligned}$$

$$173.15\,\mathrm{K} \leq T \leq 373.15\,\mathrm{K} \qquad (2)$$

$$s_{\mathrm{rel}} < \pm 0.5\% \quad (-100 \text{ to } -50\,^{\circ}\mathrm{C}) \qquad (3)$$

$$s_{\mathrm{rel}} < \pm 0.3\% \quad (-50 \text{ to } -0\,^{\circ}\mathrm{C}) \qquad (4)$$

$$s_{\mathrm{rel}} < \pm 0.005\% \quad (-0 \text{ to } -100\,^{\circ}\mathrm{C}) \qquad (5)$$

Where:

e_{w} : saturation vapor pressure (Pa)

T : temperature (K)

s_{rel} : relative standard deviation

Equation 1 is equation 7 of [2, Sonntag (1994)], “Advancements in the field of hygrometry”. Note that this equation should strictly be multiplied by an enhancement factor f since the vapor pressure of water predicted by the equation is for pure water vapor (no other gases such as oxygen or nitrogen present). However, according to [1, Green (2008)], the inaccuracy introduced by neglecting f is “less than 0.5 percent, except at elevated pressures”. Therefore, f is “usually neglected for engineering calculations”. If f were to be taken into account, then equations for $f(P,T)$ in “meteorlogical temperature and pressure range” are provided in [2, Sonntag (1994)].

Equations 3, 4, and 5 are from [2, Sonntag (1994)] and indicate the different relative standard deviations for $\ln(e_\mathrm{w}(T))$ that apply for different temperature ranges. s_rel values limit the accuracy of calculations made using $\ln(e_\mathrm{w}(T))$ as input.

1.2.2 Saturation vapor pressures of moist air

$$e'_\mathrm{w}(t) = f_\mathrm{w}(p,t)\, e_\mathrm{w}(t) \tag{6}$$
$$e' = f_\mathrm{w}(p,t_\mathrm{d})\, e_\mathrm{w}(t_d) \tag{7}$$
$$e' = f_\mathrm{w}(p,t_\mathrm{d})\, e \tag{8}$$

Where:

$e'_\mathrm{w}(t)$: saturation vapor pressure of water in moist air
$f_\mathrm{w}(p,t)$: enhancement factor as a function of p and t
$e_\mathrm{w}(t)$: saturation vapor pressure of water (e.g. no O_2 or N_2)
e' : vapor pressure of water in moist air
e : vapor pressure of water in the pure phase
p : total pressure
t : temperature

Equations 6 and 7 are equations 18 and 20 of [2, Sonntag (1994)]. Equation 8 is the result of substituting equation 23 into equation 7.

The enhancement factor f_w is required when taking into account the fact that the vapor pressure of a gas that contains only water molecules is different from the vapor pressure of water molecules that are mixed with other components in a gas mixture. e_w is strictly the saturation vapor pressure of water in systems where only water is present; introduction of air (i.e. oxygen, nitrogen, etc.) into the system also introduces an error in calculations in which the saturation vapor pressure of the water component of the gas mixture is desired. According to [1, Green (2008)], the correction factor needed to compensate for this error (i.e. f_w) is "typically less than 0.5 percent, except at elevated pressures". For purposes of this article, f_w will be taken into account; see equation 9.

Equation 7 indicates that the vapor pressure of water in moist air e' may be calculated as a function of only upon the system's total air pressure p and the system's water dew point temperature t_d.

Equation 8 is equation 7 but with equation 23 applied. Equation 8 encapsulates the idea that a non-saturated mixture of water vapor and air has a water vapor pressure that is the same as that of the mixture when cooled to its dew point temperature and total pressure is held constant.

1.2.3 Enhancement factor *f*

$$\begin{aligned} f_\mathrm{w}(p,t) &= 1 + \frac{10^{-4}\, e_\mathrm{w}(t)}{273+t}\left[(38 + 173\exp(^{-t}/_{43}))\left(1 - \frac{e_\mathrm{w}(t)}{p}\right)\right. \\ &\quad \left. + (6.39 + 4.28\exp(^{-t}/_{107}))\left(\frac{p}{e_\mathrm{w}(t)} - 1\right)\right] \end{aligned} \tag{9}$$

Where:

f_w : enhancement factor for water
p : total pressure (hPa)
t : temperature (°C)
e_w : saturation vapor pressure with respect to water (hPa)

Equation 9 is equation 22 of [2, Sonntag (1994)]. It is meant to be used with equations 6 and 7.

If t is neglected, an approximation for f_w as only a function of pressure can be made. For this, [2, Sonntag (1994)] offers two options:

$$f(p) \cong f_\mathrm{w}(p,t) \cong 1.0016 + 3.15 \times 10^{-6}\, p - \frac{0.074}{p} \tag{10}$$

Where:

f : enhancement factor
f_{w} : enhancement factor for water
p : total pressure (hPa)

Equation 10 is equation 24 in [2, Sonntag (1994)], according to which has a standard deviation of $<\pm 0.000013$ for the temperature range -50 to $+70$°C.

1.2.4 Definition of relative humidity

$$U_{\mathrm{w}} = \frac{e'}{e'_{\mathrm{w}}(p,t)} \times 100\% \tag{11}$$

Where:

U_{w} : relative humidity (%)
e' : vapor pressure of water in an air-water system
e'_{w} : saturation vapor pressure of water in an air-water system

This equation is provided as equation 31 in [2, Sonntag (1994)]]. An analogous equation is presented by [1, Green (2008)] on page 12-4 as "$\mathrm{RH} = 100\, p / p_s$"; in this context, p stands for "partial pressure of vapor in a gas-vapor mixture" and p_s is p when the system is fully saturated with vapor (i.e. water).

This work will continue to use U as the symbol for relative humidity.

1.2.5 Interconversion from vapor pressure p to absolute humidity Y

$$Y = \frac{p\, M_v}{(P - p)\, M_g} \tag{12}$$

Where:

Y : absolute humidity $\left(\frac{\text{kg of water}}{\text{kg of dry air}}\right)$
p : vapor pressure of vapor (Pa)
P : total pressure (Pa)
M_v : molecular weight of vapor $\left(\frac{\text{g}}{\text{mol}}\right)$
M_g : molecular weight of gas $\left(\frac{\text{g}}{\text{mol}}\right)$

This equation is adapted from an equation in Table 12-2 of [1, Green (2008)] "Interconversion Formulas for a General Gas-Vapor System". The equation is genearlized for systems besides that of air-water. For purposes of this article, water is the vapor and air is the gas.

Equation 12 is analogous to equation 28 of [2, Sonntag (1994)] which is equation 13 below:

$$r = \frac{\varepsilon\, e'}{p - e'} \tag{13}$$

$$\varepsilon = \frac{M_{\mathrm{v}}}{M_{\mathrm{a}}} = 0.62198 \pm 0.00002 \tag{14}$$

Where:

r : mixing ratio of moist air $\left(\frac{\text{kg water vapor}}{\text{kg dry air}}\right)$
ε : ratio of molar mass of dry air and water vapor
e' : vapor pressure of water (Pa)
p : total pressure (Pa)
M_{v} : molar mass of water vapor $\left(\frac{\text{kg}}{\text{mol}}\right)$
M_{a} : molar mass of dry air $\left(\frac{\text{kg}}{\text{mol}}\right)$

Equation 14 is equation 6 of [2, Sonntag (1994)].

1.2.6 Interconversion from volumetric humidity Y_v to absolute humidity Y

$$Y = \frac{M_v}{M_g\left(\frac{PM_v}{Y_v RT} - 1\right)} \tag{15}$$

$$Y_{\text{real}} = \frac{M_v}{M_g\left(\frac{PM_v}{Y_v ZRT} - 1\right)} \tag{16}$$

$$R = 8.314\,462\,618\,153\,54\,\frac{\text{J}}{\text{mol}\cdot\text{K}} \tag{17}$$

$$Y_v = C_2 \cdot d_\text{v} \tag{18}$$

$$C_2 = \left(\frac{\text{g}}{\text{m}^3}\right)\left(\frac{\text{kg}}{1000\,\text{g}}\right) \tag{19}$$

$$= 10^{-3}\left(\frac{{}^{\text{kg}}/_{\text{m}^3}}{{}^{\text{g}}/_{\text{m}^3}}\right) \tag{20}$$

Where:

Y : absolute humidity, ideal gas $\left(\frac{\text{kg of water}}{\text{kg of dry air}}\right)$

Y_{real} : absolute humidity $\left(\frac{\text{kg of water}}{\text{kg of dry air}}\right)$

Y_v : volumetric humidity $\left(\frac{\text{kg of water}}{\text{m}^3\text{ of dry air}}\right)$

P : total pressure (Pa)

M_v : molecular weight of vapor $\left(\frac{\text{g}}{\text{mol}}\right)$

M_g : molecular weight of gas $\left(\frac{\text{g}}{\text{mol}}\right)$

R : gas constant $\left(\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)$

T : temperature (K)

d_v : volumetric humidity $\left(\frac{\text{g}}{\text{m}^3}\right)$

C_2 : unit consistency constant $\left(\frac{{}^{\text{kg}}/_{\text{m}^3}}{{}^{\text{g}}/_{\text{m}^3}}\right)$

Equation 15 is from [1, Green (2008)] that interconverts volumetric humidity Y_v (mass water per volume air) to absolute humidity Y (mass water per mass dry air). Equation 15 is adapted into equation 16 by reintroducing compressibility Z since the equation 15 was derived assuming an ideal gas while, in contrast, this work is taking compressibility into account. Equation 18 permits use of the volumetric humidity d_v calculated by equation 32. Equation 17 defines the universal gas constant R according to the 2019 redefinition of SI base units.

1.2.7 Derivation of dewpoint temperature t_d from p

$$t_\text{d} = 13.715\,y + 8.4262 \times 10^{-1}\,y^2 + 1.9048 \times 10^{-2}\,y^3 + 7.8158 \times 10^{-3}\,y^4 \tag{21}$$

$$y = \ln\left(\frac{e}{611.213\,\text{Pa}}\right) \tag{22}$$

$$e = e_\text{w}(t_\text{d}) \tag{23}$$

$$T_\text{d} = 273.15 + t_\text{d} \tag{24}$$

Where:

t_{d} : dew point or "saturation temperature" (°C)
T_{d} : dew point or "saturation temperature" (K)
$e_{\mathrm{w}}(t_{\mathrm{d}})$: saturation vapor pressure of water at the dew point (Pa)
e : vapor pressure of water (Pa)
y : substitution variable used to simplify Equation 21.

Equations 21 and 22 are adapted from equations 10 and 9 in [2, Sonntag (1994)] respectively. Equation 23 is part of equation 9 of [2, Sonntag (1994)].

To explain, if an air-water gas system is cooled at constant pressure to the dew point temperature t_{d}, then liquid water will begin to condense from the gas. Equations 21 and 22 model the relationship between t_{d} and e; if one is known then the other must have a certain value.

1.2.8 Compressibility factor for dry air

$$Z_{\mathrm{a}} \cong 1-(70-t)\,p\times 10^{-8} \tag{25}$$

Where:

Z_{a} : compressibility for dry air (–)
t : temperature (°C)
p : total pressure (hPa)

Equation 25 is taken from equation 3 in [2, Sonntag (1994)] which also indicates that "Z is very near to Z_{a}", implying the equation may be used in case actual values for the actual compressibility of moist air Z are unavailable. Extremely accurate values of Z are typically not available since Z is a function of the mass composition of the gas mixture as well as pressure and temperature; variations in the water content of an air-water mixture changes Z slightly.

Compressibility Z is required to calculate the volumetric humidity d_{vw} in equations 26 and 27. For the purposes of this article, equation 25, which is just a function of t and p, will be used to estimate Z.

1.2.9 Saturation value of volumetric humidity d_{vw}

$$d_{\mathrm{vw}} = \frac{f_{\mathrm{w}}(p,T)\,e_{\mathrm{w}}(T)\times 10^5}{Z\,R_{\mathrm{v}}\,T} \tag{26}$$

$$d_{\mathrm{vw}} = \frac{C_1\,f_{\mathrm{w}}(p,T)\,e_{\mathrm{w}}(T)}{Z\,R_{\mathrm{v}}\,T} \tag{27}$$

$$C_1 = \left(\frac{100\,\mathrm{Pa}}{\mathrm{hPa}}\right)\left(\frac{1}{\mathrm{Pa}}\cdot\frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\frac{1}{\mathrm{N}}\cdot\frac{\mathrm{kg{\cdot}m}}{\mathrm{s}^2}\right)\left(\frac{\mathrm{J}}{1}\cdot\frac{\mathrm{s}^2}{\mathrm{kg{\cdot}m}^2}\right)\left(\frac{1000\,\mathrm{g}}{\mathrm{kg}}\right) \tag{28}$$

$$= 10^5\,\frac{\frac{\mathrm{g}}{\mathrm{m}^3}}{\frac{1}{\mathrm{hPa}}\cdot\frac{\mathrm{J}}{\mathrm{kg{\cdot}K}}\cdot\frac{\mathrm{K}}{1}} \tag{29}$$

$$R_{\mathrm{v}} = \frac{R}{M_{\mathrm{v}}} = (461.525\pm 0.013)\,\frac{\mathrm{J}}{\mathrm{kg{\cdot}K}} \tag{30}$$

Where:

d_{vw} : saturation value of volumetric humidity for water-saturated system $\left(\frac{\mathrm{g}}{\mathrm{m}^3}\right)$
f_{w} : enhancement factor as a function of p and T for water-saturated system (–)
e_{w} : saturation vapor pressure of water for water-saturated system (hPa)
Z : compressibility factor for moist air (–)
R : molar gas constant $\left(\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}}\right)$
R_{v} : specific gas constant for water vapor $\left(\frac{\mathrm{J}}{\mathrm{kg{\cdot}K}}\right)$

M_v : molar mass of water vapor $\left(\frac{\mathrm{kg}}{\mathrm{mol}}\right)$
T : temperature (K)
p : total pressure (Pa)
C_1 : unit consistency constant $\left(\frac{\frac{\mathrm{g}}{\mathrm{m}^3}}{\frac{1}{\mathrm{hPa}} \cdot \frac{\mathrm{J}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{K}}{1}}\right)$

Equation 26 is taken from equation 26 of [2, Sonntag (1994)]. In that work, d_vw is defined as "the saturation value of the absolute humidity with respect to water" with units " $\mathrm{g/m^3}$ " for temperatures up to 100 °C. This work will instead use the term "volumetric humidity" while reserving the term "absolute humidity" for the quantity $\frac{\text{mass of water}}{\text{mass of dry air}}$. Equation 30 is equation 2 of [2, Sonntag (1994)] where the uncertainty is "the standard deviation". Although more precise values for R and M_v may be used to calculate R_v, this work will use the definition given in [2, Sonntag (1994)] for convenience. C_1 is a unit consistency constant used in this work to clarify the units of the $\times 10^5$ factor so related equations can be made consistent.

1.2.10 Volumetric humidity d_v

$$d_\mathrm{v} = \frac{C_1\, f_\mathrm{w}(p, T_\mathrm{d})\, e_\mathrm{w}(T_\mathrm{d})}{Z\, R_\mathrm{v}\, T} \tag{31}$$

$$d_\mathrm{v} = \frac{C_1\, e'}{Z\, R_\mathrm{v}\, T} \tag{32}$$

Where:

d_v : volumetric humidity $\left(\frac{\mathrm{g}}{\mathrm{m}^3}\right)$
C_1 : unit consistency constant $\left(\frac{\frac{\mathrm{g}}{\mathrm{m}^3}}{\frac{1}{\mathrm{hPa}} \cdot \frac{\mathrm{J}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{K}}{1}}\right)$. See equation 29.
f_w : enhancement factor as a function of p and T for water-saturated system (–)
T : temperature (K)
T_d : dew point temperature (K)
p : total pressure (Pa)
e_w : saturation vapor pressure of water for water-saturated system (hPa)
e' : vapor pressure of water in moist air mixture (hPa)
Z : compressibility factor for moist air (–)
R_v : specific gas constant for water vapor $\left(\frac{\mathrm{J}}{\mathrm{kg{\cdot}K}}\right)$. See equation 30.

Equations 31 and 32 are adapted from equations 20, 26, and 32 of [2, Sonntag (1994)] and describe the volumetric humidity d_v ($\frac{\text{mass of water vapor}}{\text{volume of moist air}}$). It is analogous to the interconversion formula in Table 12-2 of [1, Green (2008)] for converting from vapor pressure p to volumetric humidity Y_v in equation 33 below:

$$Y_v = \frac{M_v\, p}{R\, T} \tag{33}$$

Where:

Y_v : volumetric humidity
M_v : molar mass of vapor (e.g. water)
p : vapor pressure (e.g. water)
R : universal gas constant
T : temperature

Note that equations such as 33 from [1, Green (2008)]] do not take enhancement factors into account. For reference, d_v is related to the definitions for absolute humidity Y and volumetric humidity Y_v by equations 15 and 18 respectively.

1.2.11 Conversion between Celsius and Kelvin

$$T = 273.15 + t \tag{34}$$

Where:

$$T : \text{temperature (K)}$$
$$t : \text{temperature (°C)}$$

2 Methodology

Create a set of python functions that each take the inputs of temperature T (K), total air pressure P (Pa), and relative humidity U_w (%) in order to each produce a single output of either absolute humidity Y ($\frac{\text{kg of water}}{\text{kg of dry air}}$), volumetric humidity ($\frac{\text{g of water}}{\text{m}^3\text{ of air}}$), or dew point t_d (°C).

2.1 Calculation explanations

2.1.1 Absolute humidity r

1. Calculate $e_\mathrm{w}(T)$, the saturation vapor pressure of water in a pure phase, by solving equation 1 for e_w. The required input is the given temperature T.
2. Calculate $f_\mathrm{w}(P,T)$, the enhancement factor for water, by solving equation 9 for $f_\mathrm{w}(P,T)$. The required input variables are the given pressure P and given temperature T. Apply equation 34 to convert between T (K) and t (°C). Note that the value of $e_\mathrm{w}(T)$ is already calculated in a previous step.
3. Calculate $e'_\mathrm{w}(P,T)$, the saturation vapor pressure of water in an air-water mixture, by solving equation 6 for $e'_\mathrm{w}(P,T)$. The required input variables will be f_w and e_w. Apply equation 34 to convert between T (K) and t (°C).
4. Calculate e', the vapor pressure of water in air, by solving equation 11 for e'. The required input variables are e'_w and the given relative humidity U_w.
5. Calculate r, the absolute humidity ($\frac{\text{kg water vapor}}{\text{kg dry air}}$), by solving equation 13 for r. The required input variables are vapor pressure of water in air e' and the given total air pressure P.

2.1.2 Volumetric humidity Y_v

1. Perform the steps in 2.1.1 to calculate the vapor pressure of water in air e' from the given relative humidity U_w.
2. Calculate Z, the compressibility constant of moist air, by approximating it with the compressibility constant for dry air Z_a, and solving equation 25 for Z_a.
3. Calculate d_v, the volumetric humidity ($\frac{\text{g water vapor}}{\text{m}^3\text{ moist air}}$), by solving equation 32 for d_v. The required input variables are e', T, and Z.
4. Calculate Y_v, the volumetric humidity ($\frac{\text{kg water vapor}}{\text{m}^3\text{ moist air}}$), by solving equation 18 for Y_v. The required input variable is d_v.

2.1.3 Dew point temperature t_d

1. Perform the steps in 2.1.1 to calculate the vapor pressure of water in air e' from the given relative humidity U_w.

2. Calculate $f_w(P, t_d)$, the enhancement factor for water, by solving equation 9 for $f_w(P, t_d)$. The required input variables are the given pressure P and temperature t_d. Apply equation 34 to convert between T_d and t_d. In the first iteration t_d is not available since $f_w(p, t_d)$ is a function of t_d. Therefore, for the first iteration, use equation 10 to set $f_w = f(P)$ where f is an approximation for meteorological conditions that is only a function of pressure. Use subsequent iterations to calculate more accurate values for t_d.

3. Calculate e, the vapor pressure of water in the pure phase, by solving equation 8 for e. The required input variables are f_w and e'.

4. Calculate y, an intermediate dew point calculation variable, by solving equation 22 for y. The required input variable is e.

5. Calculate t_d, the dew point temperature in °C, by solving equation 21 for t_d. The required input variable is y. Repeat the calculation steps until adequate convergence for t_d between iterations is reached.

2.2 Python implementation

A PYTHON package `convert/humidity.py` and an accompanying test script `test.py` have been prepared that calculate absolute humidity, volumetric humidity, and dew point temperature. The respective functions defined in are `rel_to_abs`, `rel_to_dpt`, and `rel_to_vol`. A compressed file containing these files is available at:

https://reboil.com/res/2021/exe/20211002..convert-humidity-snapshot.zip

The functions have been tested by installing them into a branch of the PIMORONI software published for use with their ENVIRO sensor product which runs on a RASPBERRY PI. The GIT branch of the software is named `master-bk` and is available at:

https://zdv2.bktei.com/gitweb/EVA-2020-02-2.git

The original GIT repository is available at:

https://github.com/pimoroni/enviroplus-python.git

The PIMORONI software contains example PYTHON code for communicating with sensors installed on the ENVIRO sensor such as the `BME280`. The absolute humidity and dew point temperature functions have been tested as of 2021-10-02.

Bibliography

[1] Robert H. Perry and Don W. Green. *Perry's chemical engineers' handbook (8th)*. McGraw-Hill, New York, 8th edition, 2008.

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